EFFECT OF PARTICLE COLLISIONS ON THE MOTION AND INTERPHASE HEAT TRANSFER IN A VERTICAL TWO-PHASE FLOW

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The mechanism of motion of a polydisperse material in a two-phase flow is largely determined by the collisions between particles of different sizes. This is of particular importance in calculating the heat and mass transfer between the solid phase and the transporting gas flow.

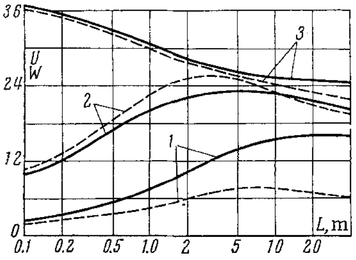

Fig. 1

However, the collision process has not been thoroughly investigated. The present authors are familiar with two studies devoted to a theoretical investigation of the motion of a multi-fraction material in a stabilized flow with absolutely elastic collisions between particles [1, 2]. Correctly assuming that in the presence of collisions the velocities of particles of any size assume values in a certain range, the authors cited postulate that only particles with minimum (if collision leads to an increase in particle velocity) or maximum (otherwise) velocity in the corresponding range can participate in the collisions. This is a highly idealized model.

The method of solving the equation of motion of the solid phase proposed in [1] is based on replacement of the actual polydisperse material by a certain conventional single-fraction material consisting of so-called equivalent particles, whose velocity (and diameter) is determined from the conditions of equality of the true and flow concentrations of the actual and conventional dispersed materials. It is assumed that the equivalent particles represent the center of gravity of the disperse material in the flow, so that for these particles the interactions with smaller and larger particles are in balance. On this basis the author proposes to replace the action of the entire system of particles of polydisperse material on a particular particle by the action of equivalent particles of corresponding concentration. The proposed model and its mathematical description are debatable. In the section that follows another method of solving the problem is proposed.

§1. Laws of particle collision in a two-phase flow. We shall consider the simple case when the dispersed material consists of spherical particles of two different sizes. We assume that particles with any velocities in the ranges of possible velocity may participate in a collision and that collisions take place only between particles of different sizes. Moreover, we shall not consider turbulent fluctuations of the gas flow. We assign the subscript 1 to the larger particles, and the subscript 2 to the smaller ones.

We shall consider a section dL of the flow large enough to accommodate a considerable number of particles of both fractions, but small enough compared with the scale of variation of the velocities and concentrations of the disperse material. In the time interval $d\tau$ a certain large particle collides with those small particles whose centers lie at the beginning of the interval $d\tau$ within a cylinder whose base is a circle of diameter $\delta_1 + \delta_2$ and whose height is equal to $(u_2 - u_1)d\tau$ (here δ is the particle diameter, and u is the vertical component of its velocity). Let the true volume concentration of small particles on the section dL be equal to $\beta_2 = w\beta_2'/u_2$ [3], where w is the gas flow velocity, and β' is the flow volume concentration. Since under actual conditions the volume concentration of solids in the flow is low, it may be assumed that collisions in which more than two particles participate are improbable [4], and only pair collisions need be considered. Accordingly, the mean free time for a large particle

$$\Delta\tau_1 = \frac{2}{3}\frac{\delta_2^3 u_2}{(\delta_1+\delta_2)^2 \beta_2' (u_2 - u_1) w}\,. \qquad (1.1)$$

A similar expression can be obtained for the small particles.

Calculations show that the particles in the flow experience very frequent collisions. Thus, at $\delta_1 = 1$ mm, $\delta_2 = 0.1$ mm, $\beta_2 = 4 \cdot 10^{-3}$ m^3/m^3, $\beta_1 = 2 \cdot 10^{-3}$ m^3/m^3, $u_2 - u_1 = 5$ m/sec, a large particle experiences more than 35 thousand collisions per second, and a small particle about 20 thousand. Accordingly, in the first approximation we can replace the discrete interaction of the fractions by a certain continuously acting force

$$F_i = m_i \Delta u_i \,/\, \Delta\tau_i\,. \qquad (1.2)$$

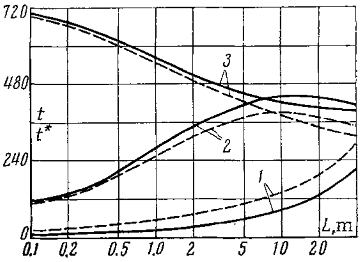

Fig. 2

Here, Δu is the change in particle velocity due to a single collision, and m is the particle mass. In this case, it is possible to consider not ranges of values of the velocity of the particles of each fraction, but certain mean values.

For an absolutely elastic glancing collision between particles with absolutely smooth surfaces the average change in the vertical component of the velocity of a large particle [1]

$$\Delta u_1 = (u_2 - u_1)\, m_2 \,/\, (m_1 + m_2), \qquad (1.3)$$

In this case the horizontal components of the particle velocities may be disregarded [1]. It is easy to see that if the collisions are not completely elastic, Eq. (1.3) must be replaced with the following equation:

$$\Delta u_1 = \frac{1+k}{2}\,\frac{m_2}{m_1+m_2}(u_2 - u_1), \qquad (1.4)$$

where k is the coefficient of restitution, which depends on the elastic properties of the colliding bodies and their relative velocity. A similar expression can be written for the small particles.

Keeping in mind (1.1), (1.2), and (1.4), we write the equations of motion of the particles in a two-phase flow [3]:

$$\frac{du_1}{d\tau_1} = g\left[\left(\frac{w-u_1}{v_1}\right)^{n_1} - 1\right] +$$

$$+\frac{3}{4}\,\frac{(1+k)\,m_2}{m_1+m_2}\,\frac{(\delta_1+\delta_2)^2\,w\beta_2'}{\delta_2^3}\,\frac{(u_2-u_1)^2}{u_2}, \qquad (1.5)$$

$$\frac{du_2}{d\tau_1} = g\left[\left(\frac{w-u_2}{v_2}\right)^{n_2} - 1\right]\frac{u_1}{u_2} -$$

$$-\frac{3}{4}\,\frac{(1+k)\,m_1}{m_1+m_2}\,\frac{(\delta_1+\delta_2)^2 w\beta_1'}{\delta_1^3}\,\frac{(u_2-u_1)^2}{u_2}. \qquad (1.6)$$

Here, v is the critical particle velocity, n is an exponent taking values from 1 to 2 depending on the value of the Reynolds number [3]. Equations (1.5) and (1.6) have been reduced to a single independent variable, the time scale associated with the motion of the large particles.

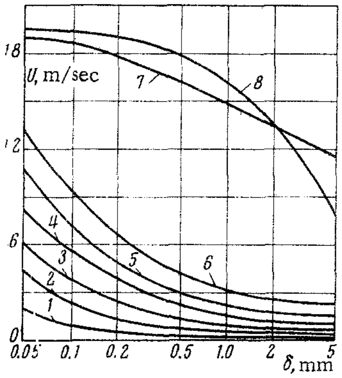

Fig. 3

In the case of an isothermal flow system (1.5), (1.6) is closed. In the presence of interphase heat transfer system (1.5), (1.6) must be supplemented with the heat transfer equations for both fractions and the heat balance equation [5]. The system of equations thus obtained can easily be solved by the method of finite differences.

As an example we have calculated the heating of a material composed of two fractions in a flow with δ_1 = 5 mm, δ_2 = 0.5 mm, w_0 = = 40 m/sec, $u_{10} = u_{20}$ = 0, t^*_0 = 800° C, $t_{10} = t_{20}$ = 0° C, k = 1 (here t is the temperature of the material, t* the temperature of the gas, and the subscript 0 indicates that the quantity relates to the initial section of the flow). Figure 1 shows the variation of the velocities u and w (m/sec), and Fig. 2 the variation of the gas temperature t* (°C) and particle temperature t (°C) along the length of the flow (continuous lines). For comparison, the broken lines represent the results of a calculation made without allowance for collisions [5]. Curves 1 in Figs. 1 and 2 relate to the large particles, 2 to the smaller particles, and 3 to the gas.

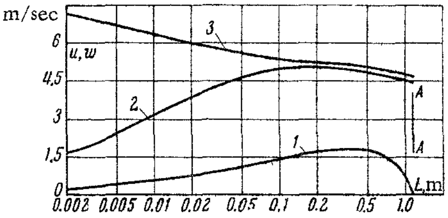

Fig. 4

It is clear from Fig. 1 that the presence in the two-phase flow of a fine fraction considerably increases the velocity of the large particles at all sections of the flow, particularly at a considerable distance from the initial section. For example, collisions lead to an increase in the velocity of the large particles on the stabilized section of the flow from 5.7 to 13 m/sec. At the same time, the reduction in the velocity of the fine particles due to collisions is inconsiderable.

Particle collisions cause the relative velocity (and hence the rate of heat transfer) to increase for small particles and decrease for large ones. Moreover, under the influence of collisions the dwell time of the particles in a certain section of the heater tube decreases markedly for large particles and increases somewhat for fine ones. These factors considerably increase the degree of nonuniformity of heating of a polydisperse material and lead to an increase in the maximum temperature to which the fine particles are heated in the flow, which is often inadmissible for technical reasons. In the presence of collisions a heater tube of considerably greater length is required to heat the coarse fraction to a given temperature, since the heating rate for coarse particles is reduced.

We shall extend the results obtained to the case of a dispersed material with a continuous particle size distribution. Let the fractional composition of the solid phase be characterized by a distribution function x(δ), while dβ' = x(δ)dδ. The problem reduces to finding the functions u = u(δ) on the stabilized and u = u(δ, L) and t = t(δ, L) on the acceleration section of the flow.

We shall consider two fractions whose particle sizes lie on the interval (δ_1, $\delta_1 + d\delta_1$) and (δ_2, $\delta_2 + d\delta_2$). The action of the second fraction on the first is given by

$$d\left(\frac{F_1}{m_1}\right) =$$

$$= \frac{3}{4}\,\frac{(1+k)\,m_2}{m_1+m_2}\,\frac{(\delta_1+\delta_2)^2\,w}{\delta_2^3}\,\frac{u_2-u_1}{u_2}\,|\,u_2 - u_1\,|\,x(\delta_2)\,d\delta_2. \quad (1.7)$$

Integrating (1.7) over the interval of variation of the particle size and keeping in mind the gravity force and aerodynamic drag, after transformations we obtain the equation of motion of the dispersed material on the stabilized section of the flow:

$$\Phi(u) \equiv g\left[\left(\frac{w-u(\delta_1)}{v_1}\right)^{n_1}-1\right]+$$

$$+\frac{3}{4}(1+k)w\times\int\limits_{\delta_{\min}}^{\delta_{\max}}\frac{\delta_1+\delta_2}{\delta_1^2+\delta_2^2-\delta_1\delta_2}\,\frac{u(\delta_2)-u(\delta_1)}{u(\delta_2)}\,|u(\delta_2)-$$

$$-u(\delta_1)|\,x(\delta_2)\,d\delta_2=0. \qquad (1.8)$$

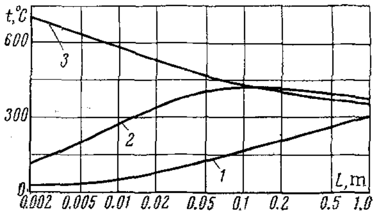

Fig. 5

Equation (1.8) can be solved quite easily on a computer by the cut-and-try method. In this case the proposed method of calculating the particle velocities does not involve a greater volume of computation than the method proposed in [1].

On the acceleration section the sum of the forces acting on a particle is obviously equal to its acceleration

$$\partial u(\delta_1,\tau_1)/\partial\tau_1=\Phi(u). \qquad (1.9)$$

In the given case the equations of interphase heat transfer [5] can also be written without modification.

The change of gas temperature on the section dL due to heat exchange with particles of the fraction (δ_1, $\delta_1 + d\delta_1$) is [5]

$$d^2t^*=-\frac{c\rho}{c^*\rho^*}\,x(\delta_1)\,d\delta_1\,dt_1. \qquad (1.10)$$

where c, ρ, c*, ρ* are the specific heat and density of the solid and gas phases, respectively. Integrating (1.10) with the initial conditions [t* = t_0*, t(δ_1, τ_1) = = t(δ_1, 0)], after transformations we obtain the heat balance equation

$$t^*=t_0^*-\frac{c\rho}{c^*\rho^*}\int\limits_{\delta_{\min}}^{\delta_{\max}}[t(\delta_1,\tau_1)-t(\delta_1,0)]\,x(\delta_1)\,d\delta_1. \qquad (1.11)$$

The system of equations obtained for the acceleration section [(1.9), (1.11), heat transfer equations] can be solved by the method of nets. In this case the required functions must be known in the initial section of the flow.

Using the above method, we calculated the motion of a dispersed material in a two-phase flow (for simplicity, isothermal conditions were considered) at $\delta_{\min}$ = 50 microns, $\delta_{\max}$ = 5 mm, x(δ) = const = = 0.4 m^{-1}, k = 1, in the initial section u(δ) = 0. Certain results of the calculation are presented in Fig. 3. Curves 1–6 relate to the acceleration section, curve 7 to the stabilized section. For comparison, we have presented the particle uniform velocity distribution without allowance for collisions (curve 8). As can be seen from Fig. 3, the small particles very quickly attain velocities close to their uniform velocities. Therefore, close to the initial section (curves 5, 6) the range of velocities of particles of the dispersed material is very broad. Subsequently, the velocity of the fine fractions varies slowly, while the velocity of the large particles continues to increase owing to collisions with fine particles and the action of the aerodynamic drag, as a result of which the range of particle velocities contracts.

We shall now examine the basic properties of the method of allowing for particle collisions described by S. I. Shabanov in [1].

The true volume concentration of the dispersed material in the flow is [3]

$$\beta=w\int\limits_{\delta_{\min}}^{\delta_{\max}}\frac{x(\delta)}{u(\delta)}\,d\delta. \qquad (1.12)$$

With account for (1.12) the velocity of the equivalent particles considered in [1] is equal to

$$u_e=\beta'\left[\int\limits_{\delta_{\min}}^{\delta_{\max}}\frac{x(\delta)}{u(\delta)}\,d\delta\right]^{-1} \qquad (1.13)$$

and their diameter δ_e can be determined from curve 7 in Fig. 3.

The size δ_* of the particles of the fraction for which the interactions with smaller and larger particles are in balance is determined from the condition that the integral on the left side of (1.8) is equal to zero. Obviously, the diameter of these particles is equal to the abscissa of the point of intersection of curves 7 and 8 in Fig. 3.

As follows from (1.12), the size δ° of the particles representing the center of gravity of the dispersed material in the flow can be determined from the equation

$$\int\limits_{\delta_{\min}}^{\delta_{\max}}\frac{x(\delta)}{u(\delta)}\,d\delta=2\int\limits_{\delta_{\min}}^{\delta^\circ}\frac{x(\delta)}{u(\delta)}\,d\delta. \qquad (1.14)$$

Thus, the three quantities δ_e, δ_*, δ° are determined from three different conditions, and in the general case the assumption made in [1] that they are equal is only a very rough approximation. For example, in the calculation mentioned above, these quantities take the following values on the stabilized section of the flow: δ_e = 2.22 mm; δ_* = 1.53 mm; δ° = 2.73 mm.

The method of solving the equations of motion of a dispersed material proposed in [1] is to apply the mean value theorem to the integral depending on the parameter:

$$\int\limits_{\delta_{\min}}^{\delta_{\max}}\frac{\delta_1+\delta_2}{\delta_1^2+\delta_2^2-\delta_1\delta_2}\,\frac{u(\delta_2)-u(\delta_1)}{u(\delta_2)}\,|u(\delta_2)-u(\delta_1)|\,x(\delta_2)\,d\delta_2=$$

$$=\frac{\delta_1+\langle\delta\rangle}{\delta_1^2+\langle\delta\rangle^2-\delta_1\langle\delta\rangle}\,\frac{u(\langle\delta\rangle)-u(\delta_1)}{u(\langle\delta\rangle)}\,|\times$$

$$\times\; u(\langle\delta\rangle)-u(\delta_1)|\,x(\langle\delta\rangle)(\delta_{\max}-\delta_{\min}). \qquad (1.15)$$

In this case it is assumed that the mean value $\langle\delta\rangle$ = const = δ_e. However, as it is easy to see from (1.15), in the general case $\langle\delta\rangle$ = = $\langle\delta\rangle(\delta_1)$, and the values of this function may vary within quite wide limits. Thus, in the above-mentioned example the quantity $\langle\delta\rangle$ takes the following values:

δ_1, mm	0.05	0.2	0.5	1	2	5
$\langle\delta\rangle$, mm	2.25	4	1.25	1.54	1.38	1.75.

§2. Motion and heat transfer of particles of variable mass in a two-phase flow. Under certain temperature and hydrodynamic conditions collisions between the particles of a multifraction material may lead to agglomeration (thermal granulation) of the dispersed material. In order to calculate this process, it is first necessary to solve two associated problems:

to establish the law of particle collision (see §1), and the laws of motion and heat transfer of particles whose mass varies along the length of the flow owing to the continual adhesion of other individual particles or aggregations.

In order to solve the second problem, we shall consider the simple case of a two-fraction material, when under given temperature and hydrodynamic conditions a certain fraction of the particle collisions leads to agglomeration.

We shall assume that the conditions adopted in §1 are satisfied. Moreover, in the first approximation we shall consider that in the process of thermal granulation the growing particles preserve their spherical shape.

We write the equation of motion of a particle of variable mass:

$$\frac{d}{d\tau_1}(m_1u_1) = -P_1 + F_{1a} + F_1 + u_2\frac{dm_1}{d\tau_1}. \qquad (2.1)$$

Here P is the weight of the particles, F_a is the aerodynamic drag, F is a continuously acting force equivalent to the action of the small particles in collisions that do not lead to agglomeration (§1). The subscripts 1 and 2 relate to the large and small particles, respectively.

In a time $\Delta\tau_1$ (1.1) the mass of a large particle increases on the average by φm_2 (φ is the fraction of collisions leading to agglomeration of the colliding particles). Since, in practice, a large particle experiences very frequent collisions (§1), we substitute for the discrete change in its mass the continuous variation

$$dm_1 / d\tau_1 = \varphi m_2 / \Delta\tau_1$$

and after transformation we obtain

$$\frac{dm_1}{d\tau_1} = \frac{\pi}{4}\varphi\rho w(\delta_1 + \delta_2)^2\beta_2'\frac{u_2 - u_1}{u_2}. \qquad (2.2)$$

In this case the expression for F_1 (§1) must be multiplied by $(1 - \varphi)$; then

$$F_1 = \frac{3}{4}(1 - \varphi)\frac{(1 + k)m_1m_2}{m_1 + m_2}\frac{(\delta_1 + \delta_2)^2 w\beta_2'}{\delta_2^3}\frac{(u_2 - u_1)^2}{u_2}. \qquad (2.3)$$

Substituting (2.2) and (2.3) as well as the value of F_{1a} from [3] into (2.1), after transformations we obtain

$$\frac{du_1}{d\tau_1} = g\left[\left(\frac{w - u_1}{v_1}\right)^{n_1} - 1\right] + \qquad (2.4)$$

$$+ \frac{3}{4}(\delta_1 + \delta_2)^2 w\beta_2'\frac{(u_2 - u_1)^2}{u_2}\left[\frac{(1 + k)(1 - \varphi)m_2}{\delta_2^3(m_1 + m_2)} + \frac{2\varphi}{\delta_1^3}\right].$$

As before, the motion of the fine particles obeys Eq. (1.6), in the right side of which the second term must be multiplied by $(1 - \varphi)$.

After substituting in (2.2) the value $m_1 = {}^1/_6\pi\rho\delta_1^3$ and after transformations, we obtain

$$\frac{d\delta_1}{d\tau_1} = \frac{\varphi w}{2}\left(\frac{\delta_1 + \delta_2}{\delta_1}\right)^2\beta_2'\frac{u_2 - u_1}{u_2}. \qquad (2.5)$$

From the assumption regarding the absence of collisions between particles of the same size it follows that the number of large particles does not vary along the length of the flow; therefore

$$\beta_1' = \frac{\rho^*}{\rho}\mu_1 = \frac{\rho^*}{\rho}\mu_{10}\left(\frac{\delta_1}{\delta_{10}}\right)^3. \qquad (2.6)$$

Since the total flow concentration is constant,

$$\beta_2' = \frac{\rho^*}{\rho}\mu_2 = \frac{\rho^*}{\rho}\left\{\mu_{20} - \mu_{10}\left[\left(\frac{\delta_1}{\delta_{10}}\right)^3 - 1\right]\right\}, \qquad (2.7)$$

where μ is the mass flow concentration.

The heat transfer equation for the fine particles is not associated with the process of agglomeration and can be used in its finished form [5]. In considering the heat transfer of the granules it is necessary to take into account the difference in temperatures of the adhering particles.

Let, at a certain section of the flow, the particles of the two fractions be at temperatures t_1 and t_2, and let the mass concentration of large particles be equal to μ_1. In this section the enthalpy of the coarse fraction is $i_1 = c\mu_1t_1$.

Differentiating this expression, we find

$$di_1 = c(\mu_1dt_1 + t_1d\mu_1). \qquad (2.8)$$

Considering that the enthalpy of the granules increases as a result both of interphase heat transfer and increase in mass, we can write

$$di_1 = c[\mu_1(dt_1/d\tau_1)^*d\tau_1 + t_2d\mu_1], \qquad (2.9)$$

where $(dt_1/d\tau_1)^*$ is the rate of variation of the temperature of the coarse particles due to heat exchange with the gas [5]. From (2.8), (2.9) we obtain

$$\frac{dt_1}{d\tau_1} = \left(\frac{dt_1}{d\tau_1}\right)^* + \frac{t_2 - t_1}{\mu_1}\frac{d\mu_1}{d\tau_1}. \qquad (2.10)$$

Finally, the heat balance equation is written in the form

$$\frac{dt^*}{d\tau_1} = -\frac{c}{c^*}\left[\mu_1\left(\frac{dt_1}{d\tau_1}\right)^* + \mu_2\frac{dt_2}{d\tau_1}\right]. \qquad (2.11)$$

By means of the system of equations (1.6), (2.4)–(2.7), (2.10), (2.11) thus obtained we can calculate the variation of the velocities, temperatures, and concentrations of both fractions, as well as the diameter (or mass) of the coarse particles along the length of the flow. This system can be solved by the method of finite differences.

As an example, we have calculated the heating and motion of a dispersed material with $\delta_{10} = 1$ mm, $\delta_2 = 0.1$ mm, $w_0 = 8$ m/sec, $\mu_{10} = \mu_{20} = 0.738$ kg/kg, $\varphi = 1$, $t_0^* = 800°$ C, $t_{10} = t_{20} = 0°$ C, $u_{10} = u_{20} = 0$.

Figures 4 and 5 show the variation of the velocity and the temperature of gas and particles of both fractions along the length of the flow. Curves 1 relate to the coarse particles (granules), 2 to the fine particles, and 3 to the gas.

In a certain section of the flow (section A-A in Fig. 4) the velocity of the large particles falls to zero, which is attributable both to a fall in the temperature of the gas flow and to an increase in the mass of the particles. In that section the coarse particles begin to fall out of the flow. Given a suitable choice of the quantity w_0 [6] the process will be stationary for any flow length, i.e., the coarse particles will be entrained to infinity.

The curves of temperature variation for the two fractions are of roughly the same nature as for nonisothermal flow without agglomeration of the particles [5]. Agglomeration leads to more rapid heating of the coarse particles, since small particles at a higher temperature participate in granule formation.

The above analysis of the process of thermal granulation of a dispersed material in the suspended state shows that this effect can provide a basis for the development of new high-intensity technological processes in many branches of industry, such as the production of cement clinker, the agglomeration of ion ore, etc. [7].

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